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Department of Space Science and Applied Physics
The Catholic University of America
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Final Report

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Abstract

The steady motion of a planing surface of moderate aspect ratio at small angles of attack is considered. Linearized theory is used with a square-root type of pressure singularity representing the flow near the leading edge. An asymptotic solution for the pressure distribution on the planing surface at large Froude number (or small β, the inverse of the Froude number) is sought. The lowest order term of the pressure distribution, obtained by setting β equal to zero, is found to be the same as the pressure distribution on the lower side of the corresponding thin wing. Higher order terms in β are obtained by an iteration process. Explicit solutions are obtained to order β^2 for rectangular planforms. Numerical results are calculated for rectangular flat plate planing surfaces of aspect ratios from 0.5 to 2.0. It is found that for large aspect ratios the lift coefficient is reduced by the gravity effect and for small aspect ratios it is increased, the dividing aspect ratio being about 1.5. The results compare reasonably well with experimental data.

I. Introduction

When a surface craft moves at low speed through water, the lift, which supports the craft on the water surface, is supplied mainly by buoyancy. If the speed of the craft is increased so that the water surface separates smoothly from the trailing edge of the craft, the craft is said to be planing, or gliding, on the water surface. During planing motion, the lift is mainly supplied by hydrodynamic forces.

An important feature of a planing motion is the splash phenomenon, which is a spray sheet thrown out ahead and sideways of the planing craft. If the angle of attack, which may be defined as a characteristic angle between the wetted surface of the planing craft and the undisturbed water surface, is small, the thickness of the splash is expected to be thin. Green (1935, 1936) made non-linear studies of two-dimensional flat plate planing at an angle of attack α with an infinite Froude number. The Froude number Fr is defined as the ratio of the inertia effect to the gravity effect, or $Fr = U^2/g\ell$, where U is the speed of the planing craft, g the gravitational acceleration and ℓ a characteristic length. In his results, the thickness of the splash was found to be proportional to α^2 for α small. Wagner (1932) studied both two-dimensional and three-dimensional planing problems at infinite Fr. In his linearized formulation, the governing equations were shown to be the same as those found in flows past thin wings, except that in planing problems the fluid under consideration is in contact with the wing on the lower side only. It is well known that in thin airfoil theory the pressure has a square-root singularity at the leading edge of the foil. Based on a local flow study, Wagner showed that to represent the splash in planing problems, the same type of pressure singularity should be used.

When the effect of gravity is considered, two-dimensional planing surfaces of various shapes have been studied by many authors (cf. Wehausen & Laitone (1960)). Recently, Maruo (1967) considered three-dimensional planing surfaces of large and small aspect ratios; however, for small aspect ratio, his method requires Fr to be very large and is not applicable to a rectangular planing surface. In the above solutions it was assumed that, as in the case of infinite Fr, the splash was a second order quantity in angle of attack and might therefore be neglected in the formulation of the linearized theory and that the pressure has a square-root singularity at the leading edge.

In this paper, we consider a steady, three-dimensional potential flow past a planing surface of moderate aspect ratio at a large Froude number. We assume that the angle of attack is small so that the problem may be linearized. In formulating the problem, we represent the planing surface by an unknown pressure distribution over the part of the water surface directly underneath it. The geometric configuration of the splash will be neglected and the pressure is assumed to have a square-root type of singularity at the leading edge of the planing surface. The perturbation potential due to this pressure distribution is expressed in the form given by Peters (1949). It involves a quadruple integral with the integrand linearly proportional to the unknown pressure. The kinematic boundary condition on the planing surface will lead to a linear integral equation for the unknown pressure distribution. To facilitate the solution of the integral equation, we expand its kernel for points nearby the planing surface, asymptotically for Fr $\longrightarrow \infty$. In this expansion the unknown pressure distribution is regarded as if it were independent of Fr. Then in a similar fashion an iteration process will yield successive terms in the pressure expansion. When Fr is set to infinity, only the lowest order term in the expansion remains. This term corresponds to the downwash

integral equation in wing theory, except that the pressure is equal to one-half of the loading on the corres[illegible] ing. The downwash integral equation is solved using a method similar to that of Watkins, Woolston and Cunningham (1959). From the solution of the downwash integral equation, the lowest order term of the pressure expansion in Fr is obtained in terms of the given "downwash". At each stage of the iteration process the same downwash integral equation has to be solved. The solution of the downwash integral equation provides each new term in the expansion of the pressure in terms of the previously obtained terms. Our solution is carried out up to terms of order Fr^{-2}. The above iteration scheme has been advanced by Cumberbatch (1958) in solving two-dimensional planing problems at high Froude number.

In this paper numerical results are given for rectangular flat plate planing surfaces having aspect ratios from 0.5 to 2.0.

2. Derivation of the Integral Equation

Consider a planing surface of moderate aspect ratio travelling at a constant velocity U over a water surface of infinite extent. The angle of attack is assumed to be small, so that linearized theory may be adopted. The water is considered to be inviscid, incompressible and of infinite depth.

We choose a set of Cartesian coordinate axes x-y-z fixed to the planing surface. The x-y plane is assumed to coincide with the undisturbed water surface, the x-axis pointing in the direction opposite to the velocity U and the z-axis in the direction opposite to the gravitational acceleration g. In this frame of reference, the fluid at infinity appears to have a uniform velocity U in the x-direction and the motion becomes steady. We choose U as our characteristic velocity. For planing problems the proper

characteristic length should be measured in the flow direction. For a three-dimensional problem another characteristic length is the span width. Therefore two Froude numbers can be defined. For convenience in numerical evaluation we choose the semi-span width as the characteristic length. Since we are dealing with moderate aspect ratios only, the choice is not important. Based on these characteristics quantities, the velocity potential $\Phi(x,y,z)$ can be written as

$$\Phi(x,y,z) = x + \phi(x,y,z), \tag{1}$$

where x denotes the velocity potential corresponding to the uniform velocity when viewed from the x-y-z system and $\phi(x,y,z)$ is the perturbation potential due to the presence of the planing surface. It is obvious that ϕ satisfies the Laplace equation

$$\Delta\phi = 0. \tag{2}$$

Let us denote the area projected by the planing surface on the x-y plane by A. If we represent the planing surface by an unknown pressure distribution on A, the linearized boundary condition of ϕ may be expressed as (see Wehausen & Laitone (1960)),

$$\frac{\partial p}{\partial x} + 2\frac{\partial^2\phi}{\partial x^2} + 2\beta\frac{\partial\phi}{\partial z} = 0, \qquad \text{on} \quad z = 0, \tag{3}$$

where the non-dimensional small parameter β is defined by

$$\beta = g\ell/U^2 = 1/Fr \tag{4}$$

and where $p = p(x,y)$ is the non-dimensional pressure. We take the ambient pressure to be zero and define p as the ratio of the pressure to the dynamic head $\frac{1}{2}\rho U^2$, where ρ is the density of the water.

From the definition of p it is clear that on the free surface, or on the part of the x-y plane outside A,

$$p = 0, \tag{5}$$

and on A, as stated before, p is unknown. The other boundary conditions on ϕ may be stated as

$$\nabla\phi = 0 \qquad \text{at} \qquad \begin{cases} x = -\infty & (6) \\ y = \pm\infty & (7) \\ z = -\infty. & (8) \end{cases}$$

Condition (6) assures us that no gravity wave will propagate upstream.

The potential which satisfies the boundary conditions (3), (5), (6), (7) and (8) is (see Peters (1949))

$$\phi(x,y,z) = \frac{1}{2\pi}\iint_A p(\xi,\eta)\,d\xi\,d\eta \left\{ \frac{1}{\pi}\operatorname{sgn}(x-\xi)\int_0^\infty \cos\nu(y-\eta)\,d\nu \cdot \right.$$

$$\cdot \int_0^\infty \frac{\tau[\beta\tau\cos\tau z - (\tau^2+\nu^2)\sin\tau z]}{(\tau^2+\nu^2)^2+\beta^2\tau^2} \exp[-\sqrt{\tau^2+\nu^2}\,|x-\xi|]\,d\tau$$

$$\left. - H(x-\xi)\int_0^\infty \left(1+\frac{\beta}{\sqrt{\beta^2+4\nu^2}}\right)\cos\nu(y-\eta)\cos[\sqrt{\beta\mu_1}(x-\xi)]\exp(\mu_1 z)\,d\nu \right\}, \tag{9}$$

where sgn (x- ξ) is the sign function of x- ξ , H(x- ξ) is the Heaviside step function and

$$\mu_1 = \frac{\beta+\sqrt{\beta^2+4\nu^2}}{2}. \tag{10}$$

If the profile of the planing surface is expressed by

$$z = f(x, y) , \tag{11}$$

and if we denote

$$W(x, y) = \frac{\partial f}{\partial x} , \tag{12}$$

the linearized kinematic boundary condition on the planing surface requires that

$$W(x, y) = \lim_{z \to 0} \frac{\partial \phi}{\partial z} \qquad \text{for } (x, y) \text{ on } A . \tag{13}$$

Equations (9) and (13) will give us an integral equation for the unknown pressure distribution on A. When we differentiate (9) with respect to z a term containing sin τz in the τ integration is produced. It can be shown that this term gives no contribution as z tends to zero. Therefore, the integral equation for the unknown pressure distribution on A becomes

$$w(x, y) = \lim_{z \to 0} \frac{1}{2\pi} \iint_A K(x-\xi, y-\eta, z; \beta) P(\xi, \eta) d\xi d\eta , \tag{14}$$

where

$$K(x, y, z; \beta) = -\frac{1}{\pi} \operatorname{sgn}(x) \int_0^\infty \cos \sqrt{} y \, d\sqrt{} \int_0^\infty \frac{\tau^2(\tau^2+\sqrt{}^2)\cos\tau z}{(\tau^2+\sqrt{}^2)^2+\beta^2\tau^2} \exp\left[-\sqrt{\tau^2+\sqrt{}^2}\,|x|\right] d\tau - H(x) \int_0^\infty \frac{2\mu_1^2}{\sqrt{\beta^2+4\sqrt{}^2}} \cos \sqrt{} y \cos(\sqrt{\beta\mu_1}\, x) \exp(\mu_1 z) \, d\sqrt{} . \tag{15}$$

In obtaining (15), (10) has been used.

3. Large Froude Number Expansion

In this section we shall expand asymptotically the kernel function $K(x-\xi, y-\eta, z; \beta)$ for large Fr, or small β, when (x,y) is on A and z is small. The method of expansion depends on the aspect ratio being of order 1 or smaller.

Let us denote the double integral occuring in K, shown in (15) by

$$I_1 = \int_0^\infty \cos\nu y\, d\nu \int_0^\infty \frac{\tau^2(\tau^2+\nu^2)\cos\tau z}{(\tau^2+\nu^2)^2+\beta^2\tau^2} \exp[-\sqrt{\tau^2+\nu^2}|x|]d\tau. \quad (16)$$

A change of variables

$$\tau = k\cos\theta \quad , \quad \nu = k\sin\theta$$

transforms (16) into

$$I_1 = \int_0^{\pi/2} \cos^2\theta\, d\theta \int_0^\infty \frac{k^3}{k^2+\beta^2\cos^2\theta} \cos(ky\sin\theta)\cos(kz\cos\theta)\exp(-k|x|)dk$$

$$= \frac{1}{2}\mathrm{Re}\int_0^{\pi/2} \cos^2\theta\, d\theta \sum_{j=1}^{2}\left\{\int_0^\infty k\exp(-k\psi_j)dk - \beta^2\cos^2\theta \int_0^\infty \frac{k\exp(-k\psi_j)}{k^2+\beta^2\cos^2\theta}dk\right\}, \quad (17)$$

where

$$\left.\begin{aligned} \psi_1 &= |x| + i(y\sin\theta + z\cos\theta), \\ \psi_2 &= |x| + i(y\sin\theta - z\cos\theta). \end{aligned}\right\} \quad (18)$$

The integrations with respect to k in (17) can all be carried out and we have

$$I_1 = \frac{1}{2}\mathrm{Re}\int_0^{\pi/2} \cos^2\theta\, d\theta \sum_{j=1}^{2} \left\{ \frac{1}{\psi_j^2} - \beta^2\cos^2\theta \left[\cos(\beta\psi_j\cos\theta)\,\mathrm{ci}(\beta\psi_j\cos\theta) \right.\right.$$

$$\left.\left. - \sin(\beta\psi_j\cos\theta)\,\mathrm{si}(\beta\psi_j\cos\theta)\right]\right\}, \tag{19}$$

where ci(x) and si(x) are the cosine and sine integrals respectively and are defined as

$$\mathrm{ci}(x) = \int_x^{\infty} \frac{\cos t}{t}\,dt = -\gamma - \ln x + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{2n(2n)!}, \tag{20}$$

$$\mathrm{si}(x) = -\int_x^{\infty} \frac{\sin t}{t}\,dt = -\frac{\pi}{2} + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-1}}{(2n-1)(2n-1)!}. \tag{21}$$

In (20), γ is Euler's constant. Under the assumption that the aspect ratio is not large ψ_1, $\psi_2 = O(1)$ and therefore I_1 may be expanded asymptotically for β small. The expansion valid up to the order of β^2 is

$$I_1 = \frac{1}{2}\mathrm{Re}\int_0^{\pi/2} \cos^2\theta\, d\theta \sum_{j=1}^{2} \left\{ \frac{1}{\psi_j^2} - \beta^2\cos^2\theta\left[-\gamma - \ln(\beta\psi_j\cos\theta) + O(\beta^2)\right]\right\}$$

$$= \frac{1}{2}\int_0^{\pi/2} \cos^2\theta\, d\theta \left\{ \frac{x^2 - (y\sin\theta + z\cos\theta)^2}{[x^2 + (y\sin\theta + z\cos\theta)^2]^2} + \frac{x^2 - (y\sin\theta - z\cos\theta)^2}{[x^2 + (y\sin\theta - z\cos^2\theta)^2]^2} \right.$$

$$+ \beta^2\cos^2\theta\left[2\gamma + 2\ln\cos\theta + 2\ln\beta + \frac{1}{2}\ln(x^2 + [y\sin\theta\right.$$

$$\left.\left. + z\cos\theta]^2)(x^2 + [y\sin\theta - z\cos\theta]^2)\right] + O(\beta^3)\right\}. \tag{22}$$

In the last expression of (22), the first two terms inside the curly brackets are independent β, and hence, gravity free; the remaining terms are dependent on β. Since

we expect to obtain the downwash integral equation in wing theory, except for a factor of 1/2, from the integral equation (14), and since when $x < \xi$ the kernel of the integral equation (14) is proportional to I_1, we expect to recover the kernel of the downwash integral equation for $x < \xi$ from the gravity free terms in (22). This means that if we set $z = 0$ in (22) we should expect a singularity in y of the form $1/y^2$ to appear after the θ integration. The meaning of this type of singularity is well known (Thwaites (1960)). Since the remaining β - dependent terms in (22) will not produce a singularity stronger than that of the gravity free terms we may, therefore, set $z = 0$ first and then integrate with respect to θ. This yields

$$\lim_{z\to 0} I_1 = \frac{\pi}{2}\Big\{\Big(1-\frac{|x|}{\sqrt{x^2+y^2}}\Big)\frac{1}{y^2} + \frac{3}{8}\beta^2\Big[\ln\frac{\beta}{4}+\gamma + \frac{7}{12} + \ln\big(|x|+\sqrt{x^2+y^2}\big) - \frac{2}{3}\Big(\frac{y}{|x|+\sqrt{x^2+y^2}}\Big)^2\Big\{1+ \frac{1}{8}\Big(\frac{y}{|x|+\sqrt{x^2+y^2}}\Big)^2\Big\}\Big] + o(\beta^3)\Big\}. \tag{23}$$

Now, let us denote the single integral term occurring in (15) by

$$I_2 = 2\int_0^\infty \frac{\mu_1^2}{\sqrt{\beta^2+4\nu^2}}\cos\nu y\cos(\sqrt{\beta\mu_1}\,x)\exp(\mu_1 z)d\nu, \tag{24}$$

where μ_1 is given by (10). We note that in (24) we have to consider only $x > 0$ as can be seen clearly from (15). Since I_2 is an even function of y we need consider only $y > 0$.

The integral I_2 may be regarded as representing part of the downwash due to a concentrated pressure point moving on the surface. It therefore contains a singularity of high order along the track line $y = \eta$ for $z = 0$. However, as pointed out by Lamb (1934), the singularity is due to the artificial nature of a concentrated pressure point and disappears for a diffused pressure. Therefore, when necessary, the η - integra-

tion will always be carried out first to remove the singularity in $(y - \eta)$ that would otherwise appear as z goes to zero. In the following expansion procedure for I_2 we shall assume z to be different from zero, when necessary, in order to avoid the appearance of the singularity in y.

By using

$$\nu = \beta \, ch\mu \, sh\mu \tag{25}$$

equation (24) may be written as

$$I_2 = \beta^2 \int_{-\infty}^{\infty} ch^4\mu \cos(\beta y \, ch\mu \, sh\mu) \cos(\beta x \, ch\mu) \exp(\beta z \, ch^2\mu) \, d\mu. \tag{26}$$

Let us add

$$\beta^2 \int_{-\infty}^{\infty} ch^4\mu \sin(\beta y \, ch\mu \, sh\mu) \sin(\beta x \, ch\mu) \exp(\beta z \, ch^2\mu) \, d\mu = 0 \tag{27}$$

to (26) to obtain

$$I_2 = \mathrm{Re}\, \beta^2 \int_{-\infty}^{\infty} ch^4\mu \exp[\beta z \, ch^2\mu + i\beta(y \, ch\mu \, sh\mu - x \, ch\mu)] \, d\mu. \tag{28}$$

The path of integration in (28) may be changed to C which runs from $-\infty + \frac{\pi}{4} i$ to $\infty + \frac{\pi}{4} i$ parallel to the Re μ-axis. Along C

$$\left.\begin{aligned} ch\mu &= \frac{1}{\sqrt{2}}(ch t + i sh t) \\ sh\mu &= \frac{1}{\sqrt{2}}(sh t + i ch t) \end{aligned}\right\}, \tag{29}$$

where t is purely real. The expansion of $\exp(-i\beta x ch u)$ into power series of $(-i\beta x ch u)$ and the substitution of (29) into (28) give

$$I_2 = \beta^2 \exp(\tfrac{1}{2}\beta z)\,\mathrm{Re}\int_{-\infty}^{\infty}\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{1}{2}\right)^{\frac{n+4}{2}}(\beta x)^n(\mathrm{sh}t - i\,\mathrm{ch}t)^{n+4}\cdot$$

$$\exp[-\tfrac{\beta}{2}(y\,\mathrm{ch}2t - iz\,\mathrm{sh}2t)]\,dt\,. \tag{30}$$

By the binomial expansion of $(\mathrm{sh}t - i\mathrm{ch}t)^{n+4}$ and by separation into real and imaginary parts we can write

$$I_2 = \beta^2\exp(\tfrac{1}{2}\beta z)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{1}{2}\right)^{\frac{n+4}{2}}(\beta x)^n\int_{-\infty}^{\infty}\Big\{\sum_{m=0,2,4,---}^{\leq n+4}(-1)^{\frac{m}{2}}\cdot$$

$$C_m^{n+4}\,\mathrm{sh}^{n+4-m}t\,\mathrm{ch}^m t\cos(\tfrac{1}{2}\beta z\,\mathrm{sh}2t) + \sum_{m=1,3,---}^{\leq n+4}(-1)^{\frac{m-1}{2}}C_m^{n+4}\cdot$$

$$\mathrm{sh}^{n+4-m}t\,\mathrm{ch}^m t\sin(\tfrac{1}{2}\beta z\,\mathrm{sh}2t)\Big\}\exp(-\tfrac{1}{2}\beta y\,\mathrm{ch}2t)\,dt\,. \tag{31}$$

It can be seen that when n is odd the integrand in (31) is an odd function of t. Therefore the only contribution is from even values of n, in which case we can write

$$\begin{aligned}\mathrm{sh}^{n+4-m}t\,\mathrm{ch}^m t &= \left(\tfrac{1}{2}\right)^{\frac{n+4}{2}}(\mathrm{ch}2t-1)^{\frac{n+4-m}{2}}(\mathrm{ch}2t+1)^{\frac{m}{2}} \quad m \text{ even} \\ &= \left(\tfrac{1}{2}\right)^{\frac{n+4}{2}}(\mathrm{ch}2t-1)^{\frac{n+3-m}{2}}(\mathrm{ch}2t+1)^{\frac{m-1}{2}}\mathrm{sh}2t \quad m \text{ odd}\,.\end{aligned} \tag{32}$$

If we set $n = 2\nu$ with $m = 2\mu$ for m even and $m = 2\mu + 1$ for m odd we get

$$I_2 = \beta^2 \exp(\tfrac{1}{2}\beta z) \sum_{\nu=0}^{\infty} \frac{1}{(2\nu)!} \left(\tfrac{1}{2}\right)^{2\nu+4} (\beta x)^{2\nu} \int_{-\infty}^{\infty} \Big\{ \sum_{\mu=0}^{\nu+2} (-1)^{\mu} C_{2\mu}^{2\nu+4} (\mathrm{ch}2t - 1)^{\nu+2-\mu} \cdot$$

$$(\mathrm{ch}2t + 1)^{\mu} \cos(\tfrac{1}{2}\beta z\,\mathrm{sh}2t) + \sum_{\mu=0}^{\nu+1} (-1)^{\mu} C_{2\mu+1}^{2\nu+4} (\mathrm{ch}2t - 1)^{\nu+1-\mu} (\mathrm{ch}2t + 1)^{\mu} \cdot$$

$$\mathrm{sh}2t \sin(\tfrac{1}{2}\beta z\,\mathrm{sh}2t) \Big\} \exp(-\tfrac{1}{2}\beta y\,\mathrm{ch}2t)\,dt. \qquad (33)$$

In (33), we write

$$(\mathrm{ch}2t \pm 1) = -\frac{2}{\beta}\left(\frac{\partial}{\partial y} \mp \frac{\beta}{2}\right), \qquad (34)$$

and noting that the integral is uniformly convergent for $y \neq 0$ we get

$$I_2 = \frac{1}{4} \exp(\tfrac{1}{2}\beta z) \sum_{\nu=0}^{\infty} \frac{1}{(2\nu)!} \left(-\frac{\beta x^2}{2}\right)^{\nu} \Big\{ \sum_{\mu=0}^{\nu+2} (-1)^{\mu} C_{2\mu}^{2\nu+4} \left(\frac{\partial}{\partial y} + \frac{\beta}{2}\right)^{\nu+2-\mu} \cdot$$

$$\left(\frac{\partial}{\partial y} - \frac{\beta}{2}\right)^{\mu} + \sum_{\mu=0}^{\nu+1} (-1)^{\mu} C_{2\mu+1}^{2\nu+4} \left(\frac{\partial}{\partial y} + \frac{\beta}{2}\right)^{\nu+1-\mu} \left(\frac{\partial}{\partial y} - \frac{\beta}{2}\right)^{\mu} \frac{\partial}{\partial z} \Big\} \int_{-\infty}^{\infty}$$

$$\cos(\tfrac{1}{2}\beta z\,\mathrm{sh}2t) \exp(-\tfrac{1}{2}\beta y\,\mathrm{ch}2t)\,dt, \qquad (35)$$

where $\frac{\partial}{\partial z}$ accounts for the factor sh2t. The integral occuring in (35) can be transformed into

$$J = 2\,\mathrm{Re} \int_0^{\infty} \exp\left[-\tfrac{1}{2}\beta \sqrt{y^2 + z^2}\,\mathrm{ch}(2t + i\delta)\right] dt, \qquad (36)$$

where $\delta = \tan^{-1}(z/y)$. For $z \leq 0$ we have $-\pi/2 \leq \delta \leq 0$ since we need consider only $y > 0$. By the change of variable

$$v = 2t + i\delta \tag{37}$$

J becomes

$$J = \mathrm{Re}\left\{\int_{i\delta}^{0} + \int_{0}^{\infty + i\delta}\right\} \exp\left(-\tfrac{1}{2}\beta\sqrt{y^2+z^2}\,\mathrm{ch}\,v\right)dv. \tag{38}$$

Since the first integral in (38) is purely imaginary and since $-\pi/2 \leq \delta \leq 0$, we have

$$J = \int_0^\infty \exp\left(-\tfrac{1}{2}\beta\sqrt{y^2+z^2}\,\mathrm{ch}\,v\right)dv. \tag{39}$$

The above integral can be integrated (Watson (1944)) to give

$$J = K_0\left(\tfrac{1}{2}\beta\sqrt{y^2+z^2}\right), \tag{40}$$

where K_o is the Bessel function of the second kind with imaginary argument, which, for β small and $\sqrt{y^2 + z^2} = 0(1)$, can be expanded as

$$K_o\left(\tfrac{1}{2}\beta\sqrt{y^2+z^2}\right) = -\ln\left(\tfrac{1}{4}\beta\sqrt{y^2+z^2}\right)\sum_{m=0}^{\infty}\frac{\left(\frac{1}{4}\beta\sqrt{y^2+z^2}\right)^{2m}}{(m!)^2}$$

$$+\sum_{m=0}^{\infty}\frac{\left(\frac{1}{4}\beta\sqrt{y^2+z^2}\right)^{2m}}{(m!)^2}\psi(m+1), \tag{41}$$

where

$$\left.\begin{aligned} \psi(m+1) &= \frac{1}{m} + \psi(m) \\ \psi(1) &= -\gamma \end{aligned}\right\} \qquad (42)$$

and γ is Euler's constant. With the aid of (40), I_2 given in (35) can now be written as

$$I_2 = \frac{1}{4}\exp\left(\tfrac{1}{2}\beta z\right)\sum_{\nu=0}^{\infty}\frac{1}{(2\nu)!}\left(-\tfrac{1}{2}\beta x^2\right)^{\nu}\left\{\sum_{\mu=0}^{\nu+2}(-1)^{\mu}C_{2\mu}^{2\nu+4}\left(\frac{\partial}{\partial y}+\frac{\beta}{2}\right)^{\nu+2-\mu}\cdot\right.$$

$$\left.\left(\frac{\partial}{\partial y}-\frac{\beta}{2}\right)^{\mu}+\sum_{\mu=0}^{\nu+1}(-1)^{\mu}C_{2\mu+1}^{2\nu+4}\left(\frac{\partial}{\partial y}+\frac{\beta}{2}\right)^{\nu+1-\mu}\left(\frac{\partial}{\partial y}-\frac{\beta}{2}\right)^{\mu}\frac{\partial}{\partial z}\right\}\cdot$$

$$K_0\left(\tfrac{1}{2}\beta\sqrt{y^2+z^2}\right). \qquad (43)$$

Using (41) and (42), we may expand (43) for β small as

$$I_2 = \exp\left(\tfrac{1}{2}\beta z\right)\left\{\frac{-y^2+z^2}{(y^2+z^2)^2} - \beta\left(1+\frac{1}{2}x^2\frac{\partial^2}{\partial y^2}\right)\frac{z}{y^2+z^2}\right.$$

$$-\frac{3}{8}\beta^2\left[\ln\frac{\beta}{4}+\gamma+\frac{1}{6}-\frac{y^2}{3(y^2+z^2)}+\frac{1}{2}\ln(y^2+z^2)\right.$$

$$\left.+\left(x^2\frac{\partial^2}{\partial y^2}+\frac{1}{18}x^4\frac{\partial^4}{\partial y^4}\right)\ln(y^2+z^2) + O(\beta^3)\right\}. \qquad (44)$$

The substitution of (44) and (23) into (14) gives

$$\begin{aligned} w(x,y) = {} & \frac{1}{4\pi}\iint_A \left[1 + \frac{x-\xi}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\right] \frac{p(\xi,\eta)}{(y-\eta)^2}\, d\xi\, d\eta + \frac{\beta}{2\pi} \lim_{z\to 0} \iint_A H(x-\xi)\Big[\\ & 1 + \frac{1}{2}(x-\xi)^2 \frac{\partial^2}{\partial y^2}\Big] \frac{z}{(y-\eta)^2+z^2}\, p(\xi,\eta)\, d\xi\, d\eta + \frac{3\beta^2}{32\pi}\Big\{ \Big(\ln\beta + \gamma + \frac{7}{12} \\ & - \ln 4\Big) \iint_A p(\xi,\eta)\, d\xi\, d\eta - \iint_A \operatorname{sgn}(x-\xi)\, G(x-\xi, y-\eta)\, p(\xi,\eta)\, d\xi\, d\eta \\ & + \iint_A H(x-\xi)\left[\ln(y-\eta)^2 - \frac{3}{2}\right] p(\xi,\eta)\, d\xi\, d\eta + \lim_{z\to 0} \iint_A H(x-\xi)\Big[2(x-\xi)^2 \frac{\partial^2}{\partial y^2} \\ & + \frac{1}{9}(x-\xi)^4 \frac{\partial^4}{\partial y^4}\Big] \ln\left[(y-\eta)^2 + z^2\right] p(\xi,\eta)\, d\xi\, d\eta \Big\} + O(\beta^3), \end{aligned} \tag{45}$$

where

$$G(x,y) = \ln\left(|x| + \sqrt{x^2+y^2}\right) - \frac{2}{3}\left(\frac{y}{|x|+\sqrt{x^2+y^2}}\right)^2 \left[1 + \frac{1}{8}\left(\frac{y}{|x|+\sqrt{x^2+y^2}}\right)^2\right]. \tag{46}$$

Now, we interchange the order of integration and differentiation with respect to y in (45) and then let z tend to zero. In doing so we must evaluate

$$\lim_{z\to 0} \int p(\xi,\eta) \frac{z}{(y-\eta)^2+z^2}\, d\eta, \tag{47}$$

where the integration is carried out over the entire span, and it is understood that z tends to zero through negative values. The result is

$$-\pi p(\xi, y). \tag{48}$$

This procedure is carried out to allow the distributed nature of the pressure to overcome the high order singularities in y that were mentioned previously in connection with I_2. This gives

$$w(x,y) = \frac{1}{4\pi}\iint_A \left[1 + \frac{x-\xi}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\right] \frac{p(\xi,\eta)}{(y-\eta)^2} d\xi d\eta - \frac{\beta}{2}\left[\int_{L.E.}^{x} p(\xi,y) d\xi\right.$$

$$\left. + \frac{1}{2}\frac{\partial^2}{\partial y^2}\int_{L.E.}^{x} (x-\xi)^2 p(\xi,y) d\xi\right] + \frac{3\beta^2}{32\pi}\left[(\ln\beta + \gamma + \frac{7}{12} - \ln 4)\cdot\right.$$

$$\iint_A p(\xi,\eta) d\xi d\eta - \iint_A \operatorname{sgn}(x-\xi)\, G(x-\xi, y-\eta)\, p(\xi,\eta)\, d\xi d\eta$$

$$- \frac{3}{2}\iint_{A_x} p(\xi,\eta) d\xi d\eta + \iint_{A_x} p(\xi,\eta)\ln(y-\eta)^2 d\xi d\eta + 2\frac{\partial^2}{\partial y^2}\iint_{A_x} (x-\xi)^2\cdot$$

$$\left. p(\xi,\eta)\ln(y-\eta)^2 d\xi d\eta + \frac{1}{9}\frac{\partial^4}{\partial y^4}\iint_{A_x} (x-\xi)^2 p(\xi,\eta)\ln(y-\eta)^2 d\xi d\eta\right] + O(\beta^3), \tag{49}$$

where $G(x-\xi, y-\eta)$ is given in (46), the lower limit L.E. indicates that the integration starts from the leading edge of the planing surface, and A_x is the part of A bounded between the leading edge and the line $\xi = x$.

4. Application to a Planing Surface of Rectangular Planform

In this section, we shall consider a rectangular planing surface having the leading edge at x = -b and the trailing edge at x = b. The integral equation shown in (49) will be solved by an interation process. The unknown pressure distribution on A is expanded into

$$p(\xi,\eta) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_{mn}\, \eta^m \sqrt{1-\eta^2}\, \ell_n(\xi), \tag{50}$$

where $\ell_n(\xi)$ is the Birnbaum expansion (1923) derived from thin airfoil theory. It is defined as

$$\ell_n(\xi) = \operatorname{ctn}\frac{\theta}{2} \qquad \text{when } n = 0$$

$$= \sin n\theta \qquad \text{when } n \geqslant 1 \tag{51}$$

where

$$\theta = \cos^{-1}(-\xi/b)\,. \tag{52}$$

However, in actual calculation, the series given in (50) will be truncated.

The iteration process starts by assuming $\beta = 0$. Equation (49) reduces to

$$W(x,y) = \frac{1}{4\pi}\iint\limits_A \left[1 + \frac{x-\xi}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\right]\frac{P(\xi,\eta)}{(y-\eta)^2}\,d\xi\, d\eta\,. \tag{53}$$

Except for a factor of 1/2, this equation is the downwash integral equation for a thin wing. Expanding p(ξ , η) in the form of (50), (53) can be solved numerically. The method used is similar to that of Watkins, Woolston and Cunningham (1959). The solution to (53) gives the first term, p_0(ξ , η), in the expansion of the pressure as

$$P_0(\xi,\eta) = \sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}^{(0)}\,\eta^m\sqrt{1-\eta^2}\,\ell_n(\xi)\,, \tag{54}$$

where M and N are constants. Using the solution for p_0 (ξ , η), equation (49), when approximated to order β may be written as

$$w(x,y) + \frac{1}{2}\beta\left[\int_{-b}^{x} p_0(\xi,y)\,d\xi + \frac{1}{2}\frac{\partial^2}{\partial y^2}\int_{-b}^{x}(x-\xi)^2 p_0(\xi,y)\,d\xi\right]$$

$$= \frac{1}{4\pi}\iint_A \left[1 + \frac{x-\xi}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\right]\frac{p(\xi,\eta)}{(y-\eta)^2}\,d\xi\,d\eta. \tag{55}$$

To solve (55), we write

$$p(\xi,\eta) = p_0(\xi,\eta) + \beta p_1(\xi,\eta). \tag{56}$$

The substitution of (56) into (55) and the use of (53) and (54) give

$$\frac{1}{2}\int_{-b}^{x} p_0(\xi,y)\,d\xi + \frac{1}{4}\frac{\partial^2}{\partial y^2}\int_{-b}^{x}(x-\xi)^2 p_0(\xi,y)\,d\xi$$

$$= \frac{1}{4\pi}\iint_A \left[1 + \frac{x-\xi}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\right]\frac{p_1(\xi,\eta)}{(y-\eta)^2}\,d\xi\,d\eta, \tag{57}$$

where the integrations on the left-hand side can all be carried out analytically. This integral equation for $p_1(\xi,\eta)$ is of the same form as (53). However, the second term on the left-hand side of (57) contains singularities of the form $(1 - y^2)^{-3/2}$ as $y \rightarrow \pm 1$, at the tips of the planing surface. This singular behaviour of the induced downwash may be due to the linearization of the problem. We assume that away from the tips the error due to linearization is smaller. Also, the numerical methods used to solve downwash integral equations of the form (53) do not use the values of the downwash $W(x,y)$ at the tips. Furthermore since the pressure vanishes at the tips

we expand $p_1(\xi,\eta)$ in the same form as $p_0(\xi,\eta)$ given in (54). We therefore write

$$p_1(\xi,\eta) = \sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}^{(1)}\,\eta^m \sqrt{1-\eta^2}\,\ell_n(\xi). \tag{58}$$

We may expect this theory to become inaccurate for cases where the influence of the tips is large, for example a rectangular plate of small aspect ratio. But, on the other hand, small aspect ratio planforms of other shapes such as delta wings will not be influenced as much by this tip effect, and we therefore expect the approach to hold even for small aspect ratios.

Using the solutions for $p_0(\xi,\eta)$ and $p_1(\xi,\eta)$, equation (49), when approximated to order β^2, becomes

$$\begin{aligned}
&w(x,y) + \tfrac{1}{2}\beta\Big[\int_{-b}^{x} p_0(\xi,y)\,d\xi + \tfrac{1}{2}\frac{\partial^2}{\partial y^2}\int_{-b}^{x}(x-\xi)^2 p_0(\xi,y)\,d\xi\Big] + \frac{3\beta^2}{32\pi}\Big[\\
&\frac{16\pi}{3}\int_{-b}^{x} p_1(\xi,y)\,d\xi + \frac{8\pi}{3}\frac{\partial^2}{\partial y^2}\int_{-b}^{x}(x-\xi)^2 p_1(\xi,y)\,d\xi - \Big(\ln\beta + \gamma + \frac{7}{12}\\
&-\ln 4\Big)\iint_A p_0(\xi,\eta)\,d\xi\, d\eta + \iint_A \operatorname{sgn}(x-\xi)\,G(x-\xi, y-\eta)\,p_0(\xi,\eta)\,d\xi\, d\eta\\
&+\frac{3}{2}\iint_{A_x} p_0(\xi,\eta)\,d\xi\, d\eta - \iint_{A_x} p_0(\xi,\eta)\ln(y-\eta)^2\,d\xi\, d\eta - 2\frac{\partial^2}{\partial y^2}\iint_{A_x}(x-\xi)^2\cdot\\
&p_0(\xi,\eta)\ln(y-\eta)^2\,d\xi\, d\eta - \frac{1}{9}\frac{\partial^4}{\partial y^4}\iint_{A_x}(x-\xi)^4 p_0(\xi,\eta)\ln(y-\eta)^2\,d\xi\, d\eta\Big]\\
&= \frac{1}{4\pi}\iint_A \Big[1 + \frac{x-\xi}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\Big]\frac{p(\xi,\eta)}{(y-\eta)^2}\,d\xi\, d\eta,
\end{aligned} \tag{59}$$

where $G(x-\xi, y-\eta)$ is given in (46). In (59), the known functions have been collected on the left-hand side and all the integrals there can be integrated analytically, except the one containing $\mathrm{sgn}(x-\xi)$, which must be integrated numerically. The integration of the integral

$$\int_{-1}^{1} p_0(\xi,\eta)\ln(y-\eta)^2 d\eta = 2\pi \sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}^{(0)} \ell_n(\xi)\Bigg[$$

$$\sum_{\mu=0}^{i(\frac{m}{2})} (-1)^{\mu} C_{\mu}^{1/2} \frac{y^{m+2-2\mu}}{m+2-2\mu} + k_m\Bigg], \tag{60}$$

where $i(m/2)$ indicates the integral part of $m/2$,

$$
\begin{aligned}
k_m &= -\frac{1}{2}\left(\frac{1}{2}+\ln 2\right) && \text{for } m = 0,\\
&= 0 && \text{for } m = \text{odd integers},\\
&= \frac{(m-1)!\left[\psi\left(\frac{m+1}{2}\right)-\psi\left(\frac{m}{2}+2\right)\right]}{2^{m+1}\left(\frac{m}{2}+1\right)!\left(\frac{m}{2}-1\right)!} && \text{for } m = \text{even integers},
\end{aligned} \tag{61}
$$

and the ψ - function is given in (46) with

$$\psi\left(\frac{1}{2}\right) = -\gamma - 2\ln 2, \tag{62}$$

makes the analytical differentiation with respect to y possible. The integral equation (59) is again the same integral equation as given in (53). To solve (59), we write

$$p(\xi,\eta) = p_0(\xi,\eta) + \beta p_1(\xi,\eta) + \beta^2\left[p_2(\xi,\eta)\ln\beta + p_3(\xi,\eta)\right]. \tag{63}$$

The substitution of (63) into (59) and the use of (55) and (56) give

$$-\frac{3}{32\pi}\iint_A P_0(\xi,\eta)\,d\xi\,d\eta = \frac{1}{4\pi}\iint_A \left[1+\frac{x-\xi}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\right]\frac{P_2(\xi,\eta)}{(y-\eta)^2}\,d\xi\,d\eta \tag{64}$$

and

$$\frac{3}{32\pi}\left[\frac{16\pi}{3}\int_{-b}^{x} P_1(\xi,y)\,d\xi + \frac{8\pi}{3}\frac{\partial^2}{\partial y^2}\int_{-b}^{x}(x-\xi)^2 P_1(\xi,y)\,d\xi - \left(\gamma+\frac{7}{12}\right.\right.$$

$$\left. -\ln 4\right)\iint_A P_0(\xi,\eta)\,d\xi\,d\eta - \iint_A \operatorname{sgn}(x-\xi)\,G(x-\xi,y-\eta)\,P_0(\xi,\eta)\,d\xi\,d\eta$$

$$+\frac{3}{2}\iint_{A_x} P_0(\xi,\eta)\,d\xi\,d\eta - \iint_{A_x} P_0(\xi,\eta)\ln(y-\eta)^2\,d\xi\,d\eta - 2\frac{\partial^2}{\partial y^2}\iint_{A_x}(x-\xi)^2\cdot$$

$$\left. P_0(\xi,\eta)\ln(y-\eta)^2\,d\xi\,d\eta - \frac{1}{9}\frac{\partial^4}{\partial y^4}\iint_{A_x}(x-\xi)^4 P_0(\xi,\eta)\ln(y-\eta)^2\,d\xi\,d\eta\right]$$

$$= \frac{1}{4\pi}\iint_A\left[1+\frac{x-\xi}{\sqrt{(x-\xi)^2+(y-\eta)^2}}\right]\frac{P_3(\xi,\eta)}{(y-\eta)^2}\,d\xi\,d\eta\,, \tag{65}$$

where $G(x-\xi\ \ , y-\eta\ \ )$ is given in (46). These two integral equations will be solved in the same way as that used to solve the integral equation (53). The solutions may be written as

$$P_2(\xi,\eta) = \sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}^{(2)}\,\eta^m\sqrt{1-\eta^2}\,\ell_n(\xi) \tag{66}$$

and

$$p_3(\xi,\eta) = \sum_{m=0}^{M} \sum_{n=0}^{N} a_{mn}^{(3)} \eta^m \sqrt{1-\eta^2}\, \ell_n(\xi) . \tag{67}$$

The substitution of (54), (58), (66) and (67) into (63) gives us the first four terms of the expansion of the pressure p(ξ , η) up to order β^2 .

5. Results for a Flat Rectangular Planing Surface

We have carried out numerical calculations for the case of a flat rectangular plate for aspect ratios, AR, varying from 0.5 to 2.0*. For large aspect ratios the expansions used in this theory are not valid, while for small aspect ratios we have noted that the theory becomes inaccurate for planforms where the tip effect predominates. In our presentation of the results we use the parameter

$$\beta_c = \frac{2gb}{U^2} = \frac{2\beta}{AR} , \tag{68}$$

where 2b is the wetted chord-length. This parameter is chosen so that the resulting Froude number, $1/\beta_c$, is based on the characteristic length in the flow direction.

We use the experimental data of Sottorf (1934) which have sufficient variation in β_c to make comparison with a theory containing the gravity effect possible.

In figure 1 we show the lift slope as a function of aspect ratio for various values of β_c . The curve β_c = 0 represents half of the lift slope for the corresponding

* The aspect ratio is defined as the ratio of the span to the wetted chord length of the planing surface.

wing wetted on both sides. Curves for β_c = 0.1 and 0.2 are chosen to show the effect of gravity and to correspond to Sottorf's values of β_c. It can be seen that the theory predicts that the effect of gravity increases the lift for aspect ratios less than about 1.5 and decreases the lift for larger aspect ratios. This tendency agrees with the theoretical results found by Maruo (1967) for the limiting cases of large and small aspect ratios and with the results of Sambraus (1938). As the aspect ratio increases in Sottorf's data the angle of attack increases and β_c decreases. The first grouping is for β_c around 0.19 with angle of attack around 4.6°. The agreement between theory and experiment is quite good. The second grouping is for β_c around 0.16 with angle of attack about 5.1° and the agreement is still good. As the angle of attack increases (groups 3, 4 and 5) the difference between the theoretical and experimental results increases but the agreement is fairly good overall. This indicates that the non-linear effect of larger angles of attack may play an important role. Generally speaking the observed values are higher than the calculated values.

In figure 2 we show the position of the center of pressure as a function of aspect ratio for various values of β_c. The data are much more scattered compared with those for the lift slope. However, agreement is reasonable. Figure 3 illustrates the chordwise pressure distribution at mid-span and at 90 percent of the span, for aspect ratios 0.5 and 1.0. The effect of gravity is to increase the pressure towards the trailing edge and towards the tips. For larger aspect ratios the increase in pressure is not as pronounced.

In conclusion, this analysis of planing surfaces of moderate aspect ratio has proved to be reasonably accurate. For smaller aspect ratios we can still obtain results from this theory but the validity of the model used is then doubtful. This

limitation is due to the singular behavior of the induced downwash near the tips, which becomes increasingly important as the aspect ratio is reduced. Further work should be directed towards the correct modelling of this tip effect.

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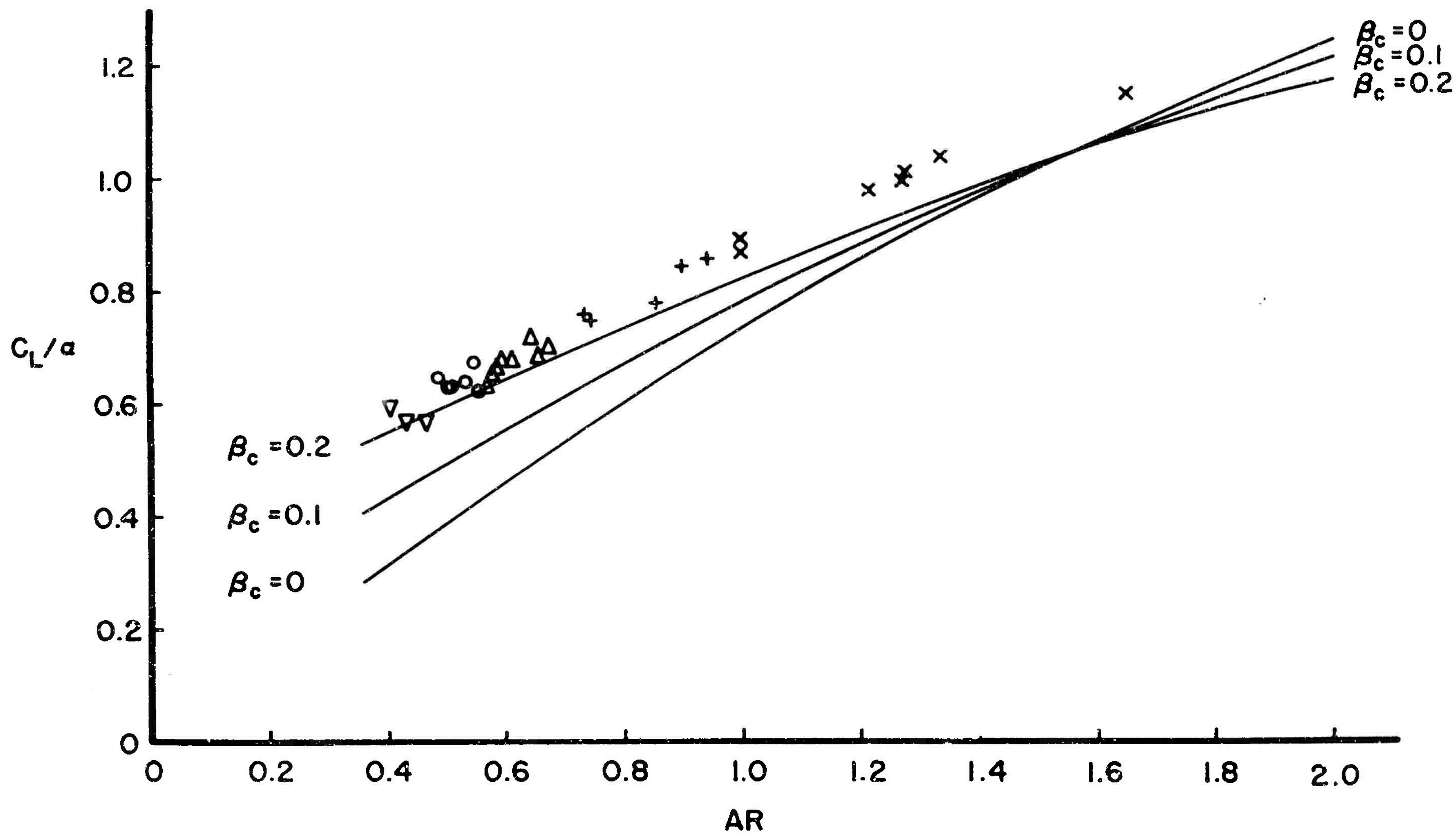

Figure 1. Lift slope C_L/a as a function of aspect ratio for various values of β_c. Present theory; ——— Experimental data is from Sottorf: ▽ $0.18 \leq \beta_c \leq 0.20$ and $4^\circ\text{-}19' \leq \alpha \leq 5^\circ\text{-}04'$; o $0.15 \leq \beta_c \leq 0.17$ and $4^\circ\text{-}43' \leq \alpha \leq 5^\circ\text{-}29'$; △ $0.12 \leq \beta_c \leq 0.14$ and $5^\circ\text{-}30' \leq \alpha \leq 6^\circ\text{-}00'$; + $0.09 \leq \beta_c \leq 0.11$ and $6^\circ\text{-}05' \leq \alpha \leq 6^\circ\text{-}53'$; × $0.06 \leq \beta_c \leq 0.08$ and $7^\circ\text{-}00' \leq \alpha \leq 8^\circ\text{-}58'$.

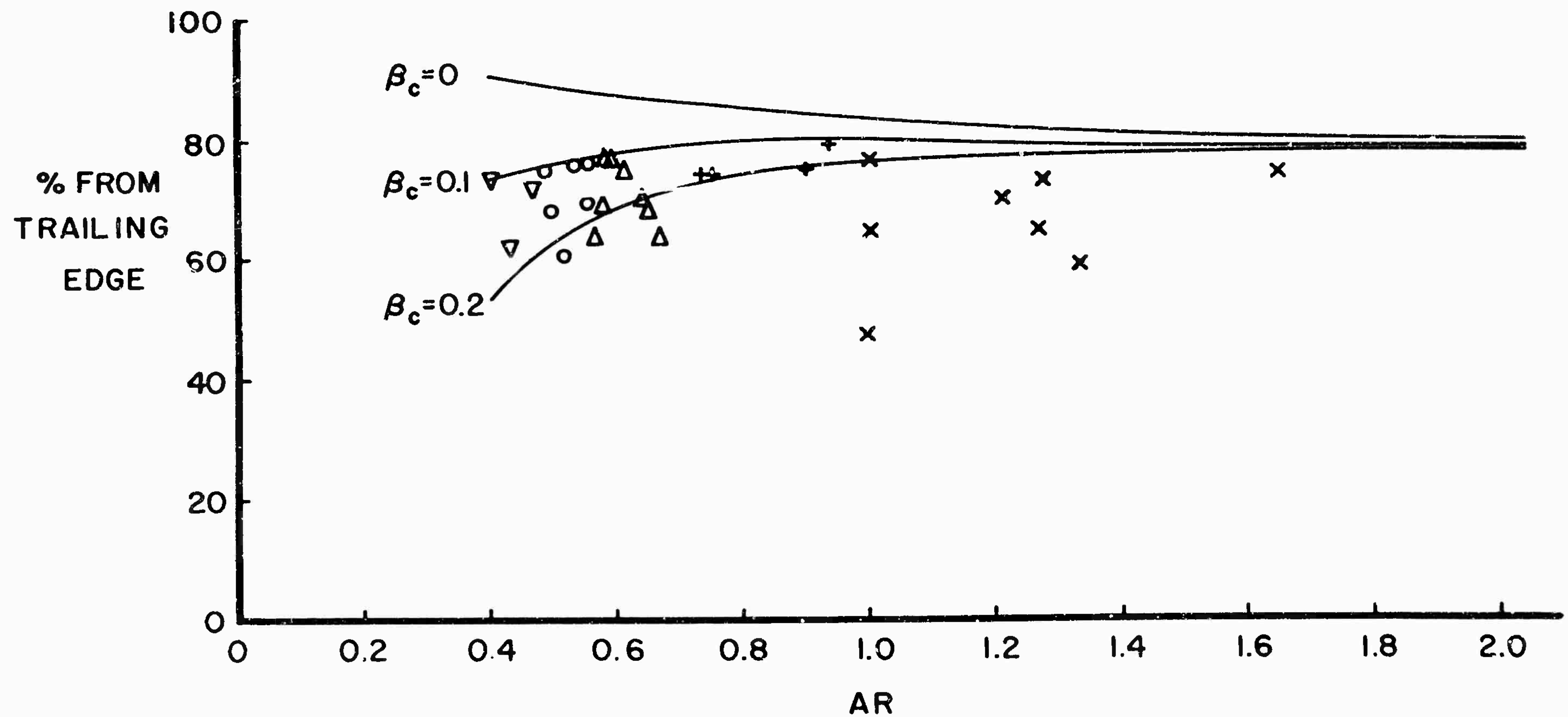

Figure 2. Position of the center of pressure as a function of aspect ratio for various values of β_c . Experimental data is from Sottorf. Symbols are the same as in figure 1.

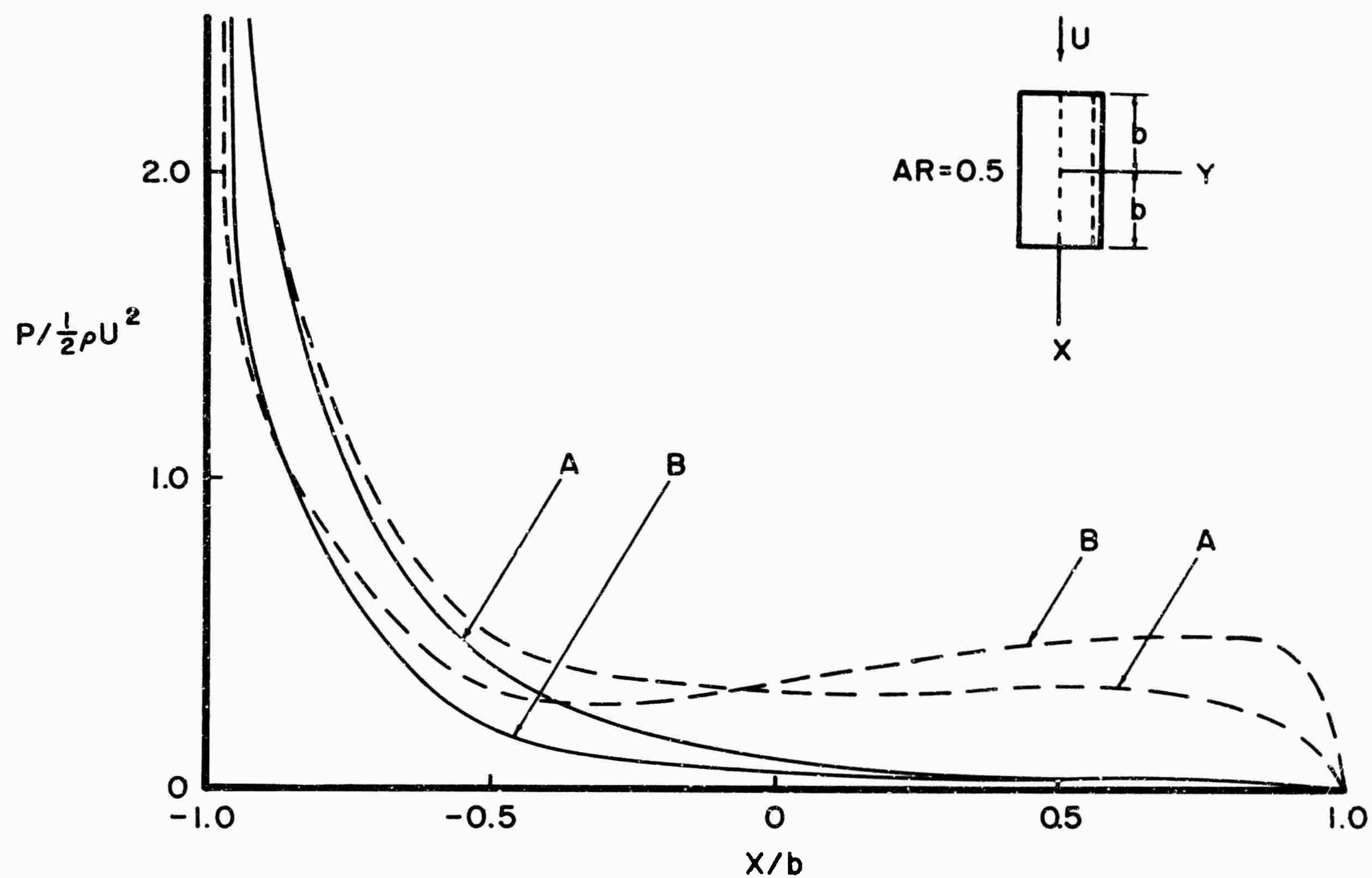

Figure 3(a). Chordwise pressure distribution at mid-span (A) and 90 percent span (B) for AR = 0.5; ——— $\beta_c = 0$. - - - - - $\beta_c = 0.2$.

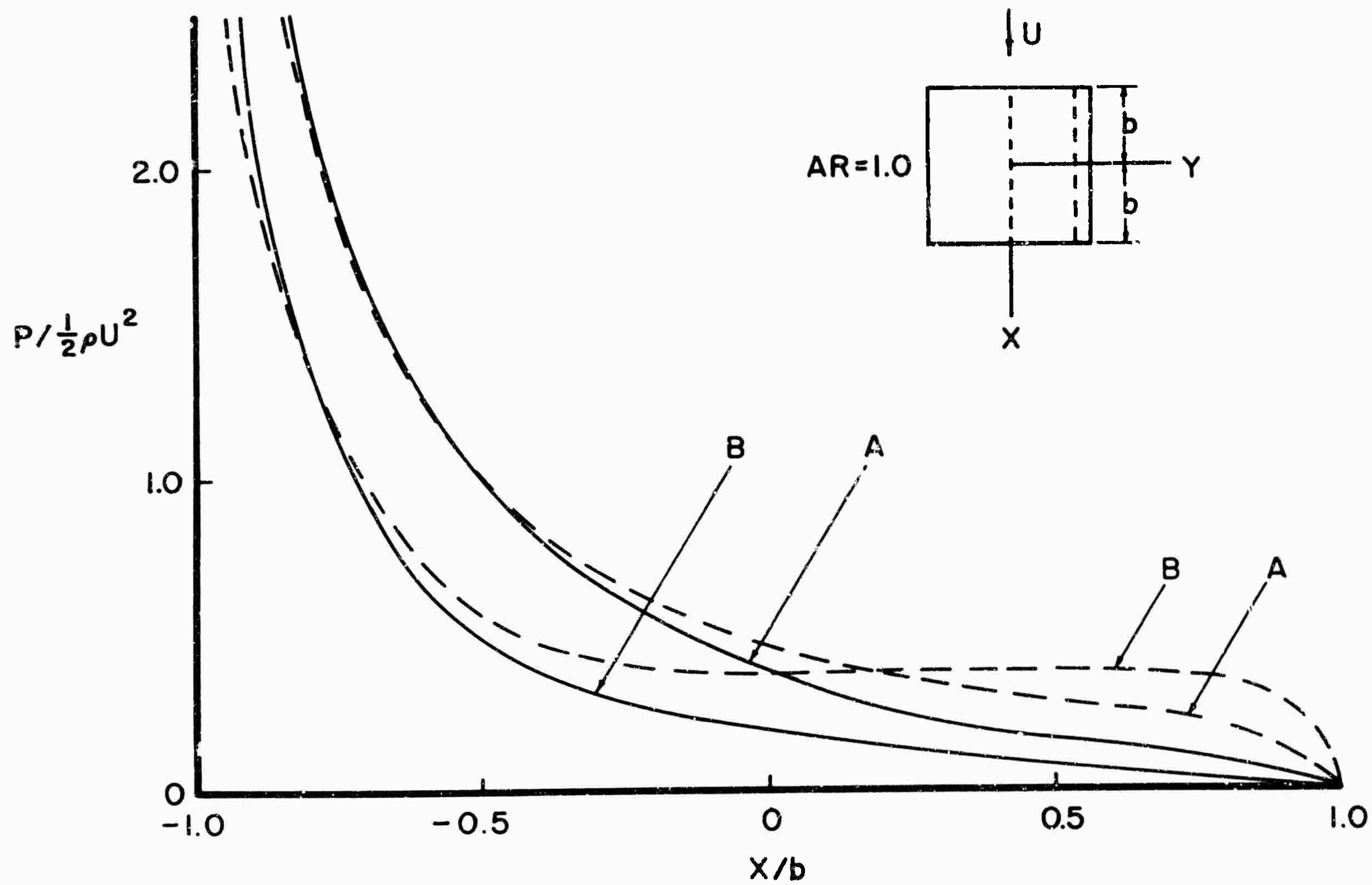

Figure 3(b). Chordwise pressure distribution at mid-span (A) and 90 percent span (B) for AR = 1.0; ——— $\beta_c = 0$, ----- $\beta_c = 0.2$.

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13. ABSTRACT

The steady motion of a planing surface of moderate aspect ratio at small angles of attack is considered. Linearized theory is used with a square-root type of pressure singularity representing the flow near the leading edge. An asymptotic solution for the pressure distribution on the planing surface at large Froude number (or small β, the inverse of the Froude number) is sought. The lowest order term of the pressure distribution, obtained by setting β equal to zero, is found to be the same as the pressure distribution on the lower side of the corresponding thin wing. Higher order terms in β are obtained by an iteration process. Explicit solutions are obtained to order β^2 for rectangular planforms. Numerical results are calculated for rectangular flat plate planing surfaces of aspect ratios from 0.5 to 2.0. It is found that for large aspect ratios the lift coefficient is reduced by the gravity effect and for small aspect ratios it is increased, the dividing aspect ratio being about 1.5. The results compare reasonably well with experimental data.

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